YASHWANTRAO CHAVAN MAHARASHTRA OPEN UNIVERSITY, NASIK

HOME ASSIGNMENT (2016-17)

V46 - MSAS 2015 CGPA Semester 2

Instructions for the Students :

1. All Questions are compulsory.
2. Write every question's answer on separate page.
3. Use of Scientific Calculator is allowed.
4. Use of Actuarial Tables is permissible.

S14021 (Actuarial Modeling - I)

Q1 Describe the advantages of the use of models in studying actuarial problems. [5]

Q2 In a game of tennis, when the score is at "Deuce" the player winning the next point holds "Advantage". If a player holding "Advantage" wins the following point that player wins the game, but if that point is won by the other player the score returns to "Deuce".

When Andrew plays tennis against Ben, the probability of Andrew winning any point is 0.6. Consider a particular game when the score is at "Deuce".

(i) Show that the subsequent score in the game can be modeled as a Markov Chain, specifying both:

(a) the state space; and

(b) the transition matrix (3)

(ii) State, with reasons, whether the chain is:

(a) irreducible; and

(b) aperiodic (2)

Q3 An investigation was carried out into the relationship between sickness and mortality in an historical population of working class men. The investigation used a three-state model with the states:

1 Healthy
2 Sick
3 Dead

Let the probability that a person in state i at time x will be in state j at time x+t be ${}_tp_x^{ij}$. Let the transition intensity at time x+t between any two states i and j be μ^{ij}_{x+t}.

Show from first principles that $\frac{\partial}{\partial t}\,{}_tp_x^{23} = {}_tp_x^{21}\mu_{x+t}^{13} + {}_tp_x^{22}\mu_{x+t}^{23}$. [5]

Q4 An internet service provider (ISP) is modeling the capacity requirements for its network. It assumes that if a customer is not currently connected to the internet ("offline") the probability of connecting in the short time interval [t, t+dt] is 0.2dt + o(dt). If the customer is connected to the internet ("online") then it assumes the probability of disconnecting in the time interval is given by 0.8dt + o(dt).

The probabilities that the customer is online and offline at time t are $P_{ON}(t)$ and $P_{OFF}(t)$ respectively.

i) Write down Kolmogorov's forward equation for $P'_{OFF}(t)$. (2)

ii) Solve the equation in part (i) to obtain a formula for the probability that a customer is offline at time t, given that they were offline at time 0. (3)

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S14022 (Actuarial Modeling - II)

Q1 A mortality investigation was held between 1 January 2011 and 1 January 2013. The following information was collected. The figures in the table below are the numbers of lives on each census date with the specified age labels.

	Date		
Age last birthday	1.1.11	1.1.12	1.1.13
48	3,486	3,384	3,420
49	3,450	3,507	3,435
50	3,510	3,595	3,540

During the investigation there were 42 deaths at age 49 nearest birthday. Estimate μ_{49} stating any assumptions that you make. [5]

Q2 As part of a clinical trial, a statistician is studying the survival rates of patients who have undergone a certain surgical procedure. Below is an extract from the statistician's data. Each patient was observed from their date of operation until their date of exit.

Patient no.	Date of operation	Date of exit	Reason for exit
1	1 February 2008	1 January 2009	Censored
2	1 April 2008	1 October 2008	Death
3	1 April 2008	1 January 2008	Censored
4	1 July 2007	1 July 2008	Censored
5	1 August 2008	1 January 2009	Censored
6	1 November 2007	1 January 2008	Death
7	1 January 2008	1 January 2009	Censored
8	1 March 2008	1 November 2008	Death
9	1 May 2008	1 November 2008	Death
10	1 June 2008	1 January 2009	Censored

You can assume that the censoring was non-informative with regard to the survival of any individual patient. Calculate the Nelson-Aalen estimate of the cumulative hazard function, $\Lambda(t)$, where t is the time in months since the operation. [5]

Q3 A life office is trying to understand the impact of certain factors on the lapse rates of its policies. It has studied the lapse rates on a block of business subdivided by:

- gender of policyholder (Male or Female)
- policy type (Term Assurance or Whole Life)
- sales channel (Internet, Direct Sales Force or Independent Financial Adviser)

The office has fitted a Cox proportional hazards model to the data and has calculated the following regression parameters:

Covariate	Regression parameter
Female	0.2
Male	0
Term Assurance	– 0.1
Whole Life	0
Internet	0.4
Independent Financial Adviser	– 0.2
Direct Sales Force	0

(i) State the gender/sales channel/policy type combination to which the baseline hazard relates. (1)

A Term Assurance is sold to a Female by an Independent Financial Adviser.

(ii) Calculate the probability that this Term Assurance is still in force after five years given that 60% of Whole Life policies bought on the Internet by Males have lapsed by the end of year five. (4)

Q4 A life insurance company has graduated its own mortality experience for term assurance business over the past 15 years against a standard table using the following equation:

$q_x = 0.94q_x^s - 0.0001$

where q_x^s is the mortality rate from the standard table.

The following is an extract from the data.

Age	Exposed to Risk	Deaths	Graduated Rate
40	24,584	14	0.000625
41	32,587	32	0.000683
42	15,784	16	0.000748
43	21,336	22	0.000823
44	25,874	24	0.000908
45	21,544	22	0.001005
46	23,967	25	0.001114
47	25,811	30	0.001239
48	26,911	28	0.001378
49	28,445	38	0.001536
50	30,205	45	0.001713

Carry out a test for overall goodness of fit of the data, using a 95% significance level. [5]

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S14023 (Casualty Actuarial Science and Time Series - I)

Q1 (i) Explain what is meant by a zero-sum two-person game. (1)

In a certain town there are two petrol stations, Station A and Station B. Both owners wish to maximise their profits. The owner of Station A has the option of charging normal prices or lower prices. The owner of the other station has the same options. If neither station lowers its prices the owner of Station A can expect to make a profit of £130 on any given day. If he lowers his prices while the other station does not, he can expect to make a profit of £170 through extra business. If he does not lower his prices but the other station does he can expect a profit of £100, and if both stations lower their prices, A can expect a daily profit of £110. Both station owners decide their pricing policy independently, and neither can alter their prices for the day once they are set.

(ii) State with reasons whether this is a zero-sum game. (1)

(iii) Describe how the owner of Station A can maximise his minimum net profit. (1)

(iv) If the figures above apply to both stations, suggest a strategy that the owners could use in order to increase their expected net daily profit. (2)

Q2 An insurer writes policies with individual excesses of Rs. 1,000 per claim. The insurer has taken out a reinsurance policy whereby the insurer pays out a maximum of Rs. 20,000 in respect of each individual claim, the rest being paid by the reinsurer.

The individual claims, gross of reinsurance and the excess, are believed to follow an exponential distribution with parameter λ. Over the last year, the insurer has gathered the following data:

- There were 8 claims which were not processed by the insurer because the loss was less than the excess.
- There were 13 claims where the insurer paid out Rs. 20,000 for each claim and the reinsurer paid the remainder.
- There were 19 other claims in respect of which the insurer paid out a total of Rs. 66,666.

Derive the log likelihood function of λ. [5]

Q3 The number, X, of claims on a given insurance policy over one year has probability distribution given by

$P(X = k) = \theta^k (1- \theta)$ $k = 0, 1, 2,\ldots\ldots$ where θ is an unknown parameter with $0 < \theta < 1$.

Independent observations $x_1,\ldots, x_n$ are available for the number of claims in the previous n years.

Prior beliefs about θ are described by a distribution with density

$f(\theta) \alpha \theta^{\alpha-1}(1- \theta)^{\beta-1}$ for some constant $\alpha, \beta > 0$.

Derive the Bayesian estimate of θ under quadratic loss and show that it takes the form of a credibility estimate:

$Z\hat{\theta} + (1 - Z)\mu$

where μ is a quantity you should specify from the prior distribution of θ. [5]

Q4 Derive an expression for the truncated mean of Normal distribution. [5]

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S14024 (Casualty Actuarial Science and Time Series - II)

Q1 a) Explain what is ruin event for a portfolio of non-life insurance policies. (2)

b) Aggregate annual claims on a portfolio of insurance policies have a compound Poisson distribution with parameter λ. Individual claim amounts have an exponential distribution with mean 1.

The insurer calculates premiums using a loading of α (so that the annual premium is (1 + α) times the annual expected claims) and has initial surplus U.

Show that if the first claim occurs at time t, the probability that this claim causes ruin is $e^{-U}.e^{-(1+\alpha)\lambda t}$ (3)

Q2 The following table shows the claim payments for an insurance company in units of £10,000:

Accident Year	Development year		
	0	1	2
2004	410	814	216
2005	575	940	
2006	814		

The inflation for a 12 month period to the middle of each year is given as follows:

2005	2006
5%	5.5%

The future inflation from 2006 is estimated to be 6% per annum.

Claims are fully run-off at the end of the development year 2.

Calculate the amount of outstanding claims arising from accidents in year 2006, using the inflation adjusted chain ladder method. [5]

Q3 There are m male drivers in each of three age groups, and data on the number of claims made during the last year are available. Assume that the numbers of claims are independent Poisson random variables.

If Y_{ij} is the number of claims for the j^{th} male driver in group i (i = 1, 2, 3; j = 1, …, m), let $E(Y_{ij}) = \mu_{ij}$ and suppose $\log(\mu_{ij}) = \alpha_i$.

i) Show that this is a generalized linear model, identifying the link function and the linear predictor. (2)

ii) Determine the log-likelihood, and the maximum- likelihood estimators of α_1 , α_2 and α_3. (3)

Q4 The following model has been suggested for representing some quarterly data with underlying seasonality.

$Y_t = \beta e_{t-4} + e_t$

where e_t is a white noise process in each case.

Determine the auto-correlation function for the model. [5]

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S14025 (Actuarial Business Economics - I)

Q1 Define price elasticity of demand**.** Explain the methods to measure price elasticity of demand. Describe the factors that affect elasticity of demand. [5]

Q2 With appropriate diagrams, explain the shapes of short run cost curves. [5]

Q3 What is meant by Non-collusive oligopoly? Describe the model of 'Kinked demand curve.' [5]

Q4 What is marketing? Explain what is meant by product–market strategy. [5]

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S14026 (Actuarial Business Economics - II)

Q1 (a) State and explain features of Public goods, Private goods and Merit goods.

(b) What is the problem of 'free rider'? [5]

Q2 Explain the comparative cost advantage theory of international trade. [5]

Q3 Describe the labour market equilibrium and indicate equilibrium level of unemployment. [5]

Q4 (a) What are the objectives of a monetary policy?

(b) Discuss Open market operations as a technique to control money supply. [5]

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